Test Bank Exercises in

CHAPTER 4

Exercise Set 4.1

1. Graph the quadratic function $f(x) = x^2 - 2x - 3$. Indicate the vertex, axis of symmetry, minimum value, and x- and y-intercepts.

2. Graph the quadratic function $f(x) = -x^2 - 2x$. Indicate the vertex, axis of symmetry, maximum value, and x- and y-intercepts.

3. Graph the quadratic function $f(x) = 3x^2 - 12x - 13$. Indicate the vertex, axis of symmetry, minimum value, and x- and y-intercepts.

4. Graph the quadratic function $f(x) = -2x^2 - 2x + 3$. Indicate the vertex, axis of symmetry, maximum value, and x- and y-intercepts.

5. Graph the quadratic function $f(x) = -3(x + 2)^2 + 1$. Indicate the vertex, axis of symmetry, maximum value, and x- and y-intercepts.

6. Graph the quadratic function $f(x) = (x - 1)(x + 4)$. Indicate the vertex, axis of symmetry, minimum value, and x- and y-intercepts.

7. An object is thrown vertically up, and its height after t seconds is given by $h(t) = -16t^2 + 144t$. Find (a) the maximum height attained by the object, and (b) the time that the object takes to hit the ground.

8. An object is thrown vertically up, and its height after t seconds is given by $h(t) = -16t^2 + 512t$. Find (a) the maximum height attained by the object, and (b) the time that the object takes to hit the ground.

9. An object is thrown vertically up with an initial velocity of 320 feet per second and its height after t seconds is given by $h(t) = -16t^2 + 320t$. Find (a) the maximum height attained by the object, and (b) the time that the object takes to hit the ground.

10. How should the length and the width of a rectangular piece of land be chosen whose boundary has the perimeter 240 yards and which encloses the maximum area.

11. Find two positive integers whose sum is 100 and whose product is maximum.

12. A businessman estimates that the daily production cost C (in dollars) to produce x number of units of a product is approximated by the function $C(x) = 0.2x^2 - 15x + 2000$. How many units should be manufactured each day to minimize the cost.

13. A businessman estimates that the net profit P is related to the advertising expenditure x (both in thousands of dollars) by $P(x) = 45 + 86x - 0.25x^2$. What advertising expenditure will maximize the profit?

14. Express the quadratic function $f(x) = 3x^2 - 6x + 1$ in the form $f(x) = a(x - h)^2 + k$. Also, find the vertex and all possible intercepts, and sketch the graph of this function.

15. The distance from the origin to the vertex of the parabola $f(x) = -.5x^2 + 2x + 4$ is
    (a) 4 (b) $2\sqrt{10}$ (c) $3\sqrt{5}$ (d) 0

16. The distance from the origin to the vertex of the parabola $f(x) = 3x^2 + 6x - 7$ is
    (a) 10 (b) $\sqrt{101}$ (c) $\sqrt{7}$ (d) 2

17. The maximum value of the quadratic function $f(x) = -4x^2 + 6x + 9$ is attained for
    (a) $x = 3/4$ (b) $x = 6$ (c) $x = 3/2$ (d) None of the above.

18. The quadratic function that has the maximum point at (–2, 1) and which passes through (0, 0) is
    (a) $f(x) = -.25(x + 2)^2 + 1$ (b) $f(x) = -3(x + 2)^2 + 1$
    (c) $f(x) = -(x - 2)^2 - 1$ (d) $f(x) = -x^2 + 4x$

19. Use your graphing calculator and sketch the graph of the function $f(x) = 3x^2 + 4x + 1$ and find the minimum value of this function.

20. Use your graphing calculator and sketch the graph of the function $f(x) = -5x^2 + 3x - 3$ and find the maximum value of this function.

Exercise Set 4.2

1. Why does the polynomial equation $x^5 - 3x - 1 = 0$ have a root inside the interval [–1, 0]?

2. Why does the polynomial equation $2x^4 - 3x^3 + x - 16 = 0$ have a root in the interval [2, 3]?

3. Find a polynomial of the smallest degree whose roots are 1/2, –1/2 and 3.

4. Find a polynomial of the smallest degree whose roots are 1, $\sqrt{3}$ and $-\sqrt{3}$.

5. Find a polynomial of the smallest degree whose roots are 2, –3 and 4.

6. Find the polynomial whose roots are 1, –1, 2 and which passes through the point (3, 9).

7. Determine the right-hand and left-hand behavior of the graph of the function $f(x) = 4x^5 - 5x^4 + 2^x - 1$.
   (a) Down to the left, down to the right.
   (b) Down to the left, up to the right.
   (c) Up to the left, up to the right.
   (d) Up to the left, down to the right.
   (e) None of the above.

8. Determine the right-hand and left-hand behavior of the graph of the function $f(x) = -3x^4 - 5x^2 - 1$.
   (a) Down to the left, down to the right.
   (b) Down to the left, up to the right.
   (c) Up to the left, up to the right.
   (d) Up to the left, down to the right.
   (e) None of the above.

9. Determine the right-hand and left-hand behavior of the graph of the function $f(x) = -x^3 + 2x - 1$.
   (a) Down to the left, down to the right.
   (b) Down to the left, up to the right.
   (c) Up to the left, up to the right.
   (d) Up to the left, down to the right.

10. For the function $f(x) = 2(x^2 - 1)(4x - 3)$, find the x- and y-intercepts and the intervals where $f(x) > 0$ and $f(x) < 0$.

11. For the function $f(x) = x^3 + x^2 - 3x - 3$, find the x- and y-intercepts and the intervals for which $f(x) > 0$ and $f(x) < 0$.

12. Graph the function $f(x) = x^3 - 3x^2 + 2x$ and find the intervals where $f(x) > 0$ and $f(x) < 0$.

13. Graph the function $f(x) = -x^3 + 5x^2 - 4x$ and find the intervals where $f(x) > 0$ and $f(x) < 0$.

14. Graph the function $f(x) = x^3 - x$. Find all the intercepts and the intervals where $f(x) > 0$ and $f(x) < 0$.

15. Graph the function $f(x) = 3x(x - 4)(x + 2)$. Find the intervals where $f(x) > 0$ and $f(x) < 0$.

16. The polynomial whose graph has intercepts at the points (1, 0), (–1, 0), (–2, 0) and (0, –2) is
   (a) $f(x) = x^3 + 2x^2 - x - 2$ (b) $f(x) = x^3 + x^2 - 2x - 2$
   (c) $f(x) = -x^3 + 2x^2 - x + 2$ (d) None of the above.

17. The polynomial whose roots are 2, –1 and 3 and whose graph passes through the point (1, 4) is
   (a) $f(x) = x^3 + 2x^2 - x - 2$ (b) $f(x) = x^3 - 4x^2 + x + 6$
   (c) $f(x) = 2x^3 + x^2 + 3x - 2$ (d) None of the above.

18. The polynomial whose roots are 1, –1, 2, –2 and whose graph passes through the point (0, 4) is
   (a) $f(x) = 4(x^4 - 5x^2 + 4)$ (b) $f(x) = x^4 - 5x^2 + 4$

(c) $f(x) = x^4 + 4x^2 - 5$ (d) None of the above.

19. Use your graphing calculator to sketch the graph of the function $f(x) = 4x^3 - 4x^2 - 9x + 9$. Locate the intercepts.

20. Use your graphing calculator to graph the function $f(x) = -2x(x - 1)(x + 2)(x - 2)(x + 3)$. Locate the intercepts.

Exercise Set 4.3

1. After dividing $x^4 - 3x^2 + 4x - 2$ by $x - 1$, the remainer is
   (a) 0 (b) $x^3 - 3x + 2$
   (c) $x^3 + 3x^2 + x - 2$ (d) None of the above.

2. When $x^3 - 3x^2 + x - 7$ is divided by $x^2 + 1$, the remainder is
   (a) -4 (b) $x - 7$ (c) $x + 7$ (d) None of these.

3. When $6x^4 + 5x^2 - 7x + 10$ is divided by $2x^2 + 1$, the remainder is
   (a) $-7x + 9$ (b) $x + 1$ (c) $2x + 4$ (d) $3x^2 + 10$

4. After dividing $x^5 - x^3 + 2$ by $x^3 - 1$, the remainder is
   (a) $x^2 + 1$ (b) $x + 2$ (c) $x^2 - x - 2$ (d) 0

5. Find the quotient $Q(x)$ and the remainder $R(x)$ when $x^3 - 6x^2 + 10$ is divided by $x^2 + 2$.

6. Find the quotient $Q(x)$ and the remainder $R(x)$ when $x^4 + x^3 - 2x^2 - x + 1$is divided by $x^2 - 1$.

7. Use the polynomial division to find the quotient and the remainder when $x^3 - 1$ is divided by $x - 1$.

8. Use the polynomial division to find the quotient and the remainder after $x^5 - 1$ is divided by $x - 1$.

9. Use the synthetic division to find the quotient $Q(x)$ and the constant remainder R when the polynomial $3x^3 + 2x^2 + 1$ is divided by $x + 1$.

10. Use the synthetic division to find the quotient $Q(x)$ and the constant remainder R when the polynomial $4x^3 + 5x^2 + 3x - 1$ is divided by $x + 2$.

11. Use the synthetic division to find the quotient $Q(x)$ and the constant remainder R when the polynomial $x^4 - 4x^3 + x^2 - 3x - 4$ is divided by $x - 4$.

12. Use the synthetic division to find the quotient $Q(x)$ and the constant remainder R when the polynomial is $5x^4 + 4x^3 + 3x^2 + 2x + 1$ is divided by $x + 1$.

13. Use the synthetic division to find the quotient $Q(x)$ and the constant remainder R when the polynomial $x^3 - 6x^2 + 12x - 8$ is divided by $x - 2$.

14. Use the synthetic division to find the quotient $Q(x)$ and the constant remainder R when the polynomial $x^3 - 64$ is divided by $x - 4$.

15. Use the synthetic division to find the quotient $Q(x)$ and the constant remainder R when the polynomial $3x^4 - 11x^3 + 6x^2 + x - 3$ is divided by $x - 3$.

16. Use the synthetic division to find the quotient $Q(x)$ and the constant remainder R when the polynomial $x^5 + 243$ is divided by $x + 3$.

17. Use the synthetic division to find the quotient $Q(x)$ and the constant remainder R when the polynomial $x^3 + x^2 + x + 1$ is divided by $x - 7$.

18. When $6x^3 - 4x^2 + 7x - 9$ is divided by $x - 3$, the quotient $Q(x)$ and the constant remainder R are
    (a) $Q(x) = 6x^2 - 10x + 61$; $R = 9$
    (b) $Q(x) = 6x^2 + 14x + 49$; $R = 138$
    (c) $Q(x) = 6x^2 + 16x - 23$; $R = 241$
    (d) None of the above.

19. When $5x^4 + 14x^3 - 19x^2 + 8x - 48$ is divided by $x + 4$, the quotient $Q(x)$ and the constant remainder R are
    (a) $Q(x) = 5x^3 + 14x + 11$; $R = 7$
    (b) $Q(x) = 5x^3 - 4x + 49$; $R = 18$
    (c) $Q(x) = 5x^3 - 6x^2 + 5x - 12$; $R = 0$
    (d) None of the above.

20. When $4x^4 - 4x^2 + 7x - 11$ is divided by $x - 2$, the quotient $Q(x)$ and the constant remainder R are
    (a) $Q(x) = 4x^2 - 7x + 11$; $R = 5$
    (b) $Q(x) = 4x^2 + 4x + 15$; $R = 19$
    (c) $Q(x) = 4x^2 + 6x - 13$; $R = 241$
    (d) None of the above.

Exercise Set 4.4

1. Use the Remainder Theorem to determine the remainder when $P(x) = 2x^3 - 3x^2 + 6x - 4$ is divided by $x + 1$.

2. Use the Remainder Theorem to determine the remainder when $P(x) = x^4 - 84$ is divided by $x - 3$.

3. Use the Remainder Theorem to determine the remainder when $P(x) = 2x^4 - 5x^3 + 7x^2 + 10$ is divided $x - 2$.

4. Use the Factor Theorem to determine whether or not $x - 4$ is a factor of $x^3 - 64$.

5. Use the Factor Theorem to decide whether or not $x + 2$ is a factor of $x^4 - x^3 - 6x^2 + 10x + 20$.

6. Use the Factor Theorem to decide whether or not $x + 5$ is a factor of $x^4 + 6x^2 - 3x + 10$.

7. Use the Factor Theorem to decide whether or not $x - 5$ is a factor of $x^4 - 5x^3 - 3x^2 + 19x + 20$.

8. Determine whether the number 3 is a zero of $f(x) = x^4 - 3x^3 - 3x^2 + 13x - 12$.

9. Determine whether the number -4 is a zero of $f(x) = x^4 + 6x^3 + 8x^2 - x - 3$.

10. Determine whether the number $3/2$ is a zero of $f(x) = 2x^3 + 5x^2 - 10x - 3$.

11. Determine whether the number $-3/4$ is a zero of $f(x) = x^4 - 3x^3 + 10x + 1$.

12. Determine whether or not $x = 5$ is a root of the equation $x^3 - 5x^2 - 4x + 20 = 0$.

13. Determine whether or not $x = 1/2$ is a root of the equation $2x^3 + x^2 + 7x + 3 = 0$.

14. Determine whether or not $x = -3/4$ is a root of the equation $4x^4 + 3x^3 + 4x^2 + 7x + 3 = 0$.

15. Find all the zeros of the polynomial $f(x) = (x - 2)^2(2x + 3)$.

16. Find all the zeros of the polynomial $f(x) = (3x + 1)^3(x - 4)$.

17. Find all the zeros of the polynomial $f(x) = 2(x - 1)(2x + 3)(x + 4)$.

18. Find all the zeros of the polynomial $f(x) = \frac{1}{2}(2x + 1)(x - 1)(3x + 2)$.

19. Determine the value of k for which $x^3 + x^2 + kx - 1$ is divisible by $x - 1$.

20. Determine the value of k for which $2x^3 + kx^2 + 10x + 8$ is divisible by $x + 2$.

Exercise Set 4.5

1. Let $z = 1 - 4i$ and let $w = 2 + 3i$. Find the sum $z + w$ and the product zw. Express your answer in the standard form $a + bi$.

2. Let $z = -5 + 7i$ and let $w = 3 - 2i$. Find the sum $z + w$ and the product zw. Express your answer in the standard form $a + bi$.

3. Let $z = 4 + 3i$ and let $w = 4 - 3i$. Find the sum $z + w$ and the product zw. Express your answer in the standard form $a + bi$.

4. Express $\frac{1 + i}{2 - 3i}$ in the standard form $a + bi$.

5. Express $\frac{6 + 7i}{2 - i}$ in the standard form $a + bi$.

6. Find the polynomial $f(x)$ of the lowest degree whose zeros are 1, –3, 3; $f(2) = -10$.

7. Find a polynomial of the lowest degree whose zeros are $\sqrt{2}, -\sqrt{2}, 4$.

8. Find a polynomial of the lowest degree whose zeros are $0, 1 - \sqrt{3}, 1 + \sqrt{3}$.

9. Find a polynomial of the lowest degree, with real coefficients, whose zeros are 3, $4i$, and $-4i$.

10. Find a polynomial of the lowest degree, with real coefficients, whose zeros are -1, $\sqrt{2}i$ and $-\sqrt{2}i$.

11. Find a polynomial of the lowest degree, with real coefficients, whose zeros are 1, $1 + i$.

12. Find a polynomial of the lowest degree, with real coefficients, whose roots are $2i$, and 2 as a root of multiplicity 3.

13. Given that –4 is a zero of the polynomial $f(x) = 2x^3 + 8x^2 - x - 4$, find the remaining zeros of $f(x)$ and express $f(x)$ as a product of linear factors.

14. Given that 1 is a zero of the polynomial $f(t) = 3t^3 - 3t^2 - t + 1$, find the remaining zeros of $f(t)$ and express $f(t)$ as a product of linear factors.

15. Given the –1 is a zero of the polynomial $f(z) = 5z^3 + 5z^2 - z - 1$, find the remaining zeros of $f(z)$ and express $f(z)$ as a product of linear factors.

16. Given that –1 and –2 are zeros of the polynomial $f(x) = 2x^4 + 6x^3 + 3x^2 - 3x - 2$, find the remaining zeros of $f(x)$ as a product of linear factors.

17. Given that 0 and 1 are the roots of the equation $x^4 - 5x^3 + 8x^2 - 4x = 0$, find the remaining roots of this equation.

18. Determine all the zeros of the polynomial function $f(x) = 2x(1 - x)(3x + 2)^2$.

19. Determine all the zeros of the polynomial function $f(x) = (x^2 + 2)(x^2 - 9)$.

20. Determine all the zeros of the polynomial function $f(x) = 2(x^2 - 4)^3(x^2 + 5)$.

Exercise Set 4.6

1. Use the Rational Zero Theorem and quotient polynomials to find all the zeros of the polynomial $f(x) = 8x^3 - 12x^2 - 2x + 3$.

2. Use the Rational Zero Theorem and the quotient polynomials to find all the zeros of the polynomial $f(x) = 4x^3 - 4x^2 - 9x + 9$, and graph the function $f(x)$.

3. Use the Rational Zero Theorem and the quotient polynomials to find all the zeros of the polynomial $f(x) = 4x^3 + 8x^2 - 3x - 18$.

4. Use the Rational Zero Theorem and the quotient polynomials to find all the zeros of the polynomial $f(x) = 6x^3 - 2x^2 - 3x + 1$. Also, use your graphing calculator to sketch the graph of the function $f(x)$.

5. Use the Rational Zero Theorem and the quotient polynomials to find all the zeros of the polynomial $f(x) = 6x^3 - 3x^2 - 2x + 1$, and graph the function $f(x)$.

6. Use the Rational Zero Theorem and the quotient polynomials to find all the zeros of the polynomial $f(x) = 9x^3 + 9x^2 - x - 1$. Also, use your graphing calculator to sketch the graph of the function $f(x)$.

7. Use the Rational Zero Theorem and the quotient polynomials to find all the zeros of the polynomial $f(x) = 12x^3 + 3x^2 - 3x - 2$.

8. Find all the zeros of the polynomial $f(x) = 3(2x - 1)(x + 1)(x - 2)$ and sketch the graph of the function $f(x)$.

9. Use the Rational Zero Theorem and the quotient polynomials to find all the zeros of the polynomial $f(x) = 4x^4 + 7x^2 - 2$.

10. Use the Rational Zero Theorem and quotient polynomials to find all the zeros of the polynomial $f(x) = 8x^4 - 14x^2 + 3$.

11. Use Descarte's Rules of Signs to analyze the nature of the roots of the equation $9x^4 - 11x^2 + 2 = 0$.

12. Use Descarte's Rules of Signs to analyze the nature of the roots of the equation $x^3 - x^2 + x - 1 = 0$.

13. Use Descarte's Rules of Signs to analyze the nature of the roots of the equation $2x^3 - 3x^2 - 2x + 3 = 0$.

14. Use Descarte's Rules of Signs to analyze the nature of the roots of the equation $x^4 + 2x^3 + 2x^2 + 2x + 1 = 0$.

15. Use Descarte's Rule of Signs, the Rational Zero Theorem and quotient polynomials to find all the roots of the equation $2x^4 + 5x^3 - x^2 + 5x - 3 = 0$.

16. Use Descarte's Rule of Signs, the Rational Zero Theorem and quotient polynomials to find all the roots of the equation $3x^4 + x^3 + x^2 + x - 2 = 0$.

17. Use Descarte's Rule of Signs, the Rational Zero Theorem and quotient polynomials to find all the roots of the equation $2x^4 + x^3 + 5x^2 + 3x - 3 = 0$.

18. Use Descarte's Rule of Signs, the Rational Zero Theorem and quotient polynomials to find all the roots of the equation $x^4 - 3x^3 + 5x^2 - 9x + 6 = 0$.

19. Use Descarte's Rule of Signs, the Rational Zero Theorem and quotient polynomials to find all the roots of the equation $2x^4 - 3x^3 - 3x - 2 = 0$.

20. Use Descarte's Rule of Signs, the Rational Zero Theorem and quotient polynomials to find all the roots of the equation $6x^4 - 22x^3 - 5x^2 - 11x - 4 = 0$.

Exercise Set 4.7

1. Write the polynomial $2x^4 + 6x^3 - 7x^2 + 3x - 4$ in the nested form.

2. Write the polynomial $3x^3 - 7x^2 - 8x + 2$ in the nested form.

3. Write the polynomial $6x^4 + 5x^3 - x^2 - x - 1$ in the nested form.

4. Write the polynomial $x^5 - 6x^3 + 3x + 2$ in the nested form.

5. Write the polynomial $2x^5 + 5x^4 - 6x^3 - 7x + 6$ in the nested form.

6. Use your calculator and the *method of successive digits* to find a root of the polynomial equation $3x^3 + 4x^2 - 9x + 1 = 0$ in the interval [0, 1] up to two decimal places.

7. Use your calculator and the *method of successive digits* to find a root of the polynomial equation $3x^3 + 4x^2 - 9x + 1 = 0$ in the interval [1, 2] up to two decimal places.

8. Use your calculator and the *method of successive digits* to find a root of the polynomial equation $x^3 - 5x^2 + 8x - 2 = 0$ in the interval [0, 1] up to two decimal places.

9. Use your calculator and the *method of successive digits* to find a root of the polynomial equation $2x^3 - 3x^2 - 7x + 1 = 0$ in the interval [0, 1] up to two decimal places.

10. Use your calculator and the *method of successive digits* to find a root of the polynomial equation $2x^3 - 3x^2 - 7x + 1 = 0$ in the interval [–2, –1] up to two decimal places.

11. Use your calculator and the *method of successive digits* to find a root of the polynomial equation $2x^3 - 3x^2 - 7x + 1 = 0$ in the interval [2, 3] up to two decimal places.

12. Use your calculator and the *method of successive digits* to find a root of the polynomial equation $x^3 + 3x^2 - 8x + 3 = 0$ in the interval [1, 2] up to two decimal places.

13. Use your calculator and the *method of successive digits* to find a root of the polynomial equation $x^3 + 3x^2 - 8x + 3 = 0$ in the interval [0, 1] up to two decimal places.

14. Us your calculator and the *method of bisection* to find a root of the polynomial equation $4x^3 - 6x^2 + 7x - 2 = 0$ in the interval [0, 1] to two decimal places.

15. Use your calculator and the *method of bisection* to find a root of the polynomial equation $x^4 - 6x^2 + 1 = 0$ in the interval [0, 1] to two decimal places.

16. Use your calculator and the *method of bisection* to find a root of the polynomial equation $x^4 - 6x^2 + 1 = 0$ in the interval [–1, 0] to two decimal places.

17. Use your calculator and the *method of bisection* to find a root of the polynomial equation $x^4 - 6x^2 + 1 = 0$ in the interval [2, 3] to two decimal places.

18. Use your calculator and the *method of bisection* to find a root of the polynomial equation $2x^3 - 6x + 7 = 0$ in the interval [–3, –2] to two decimal places.

19. Use your calculator and the *method of bisection* to find a root of the polynomial equation $3x^3 - 5x^2 + 6 = 0$ in the interval [–1, 0] to two decimal places.

20. Use your calculator and the *method of bisection* to find a root of the polynomial equation $x^3 - x^2 + 4x + 1 = 0$ in the interval [–1, 0] to two decimal places.